Lebanese American University — MEE 241 – Exam I

# Dynamics – Exam I

School of Engineering – Dep. of Industrial & Mechanical Eng.

**Name:**
**Date:** Saturday, November 10th 2011; 09:00 AM
**Location:** ENG 402
**Instructor:** Dr. Wassim Habchi
**Notes:** No documents allowed
**Value:** 25% of Total Grade
**Time:** 2 hours

$\left[\dfrac{88}{100}\right]$

## Problem I (20 points)

20

The acceleration of a particle as it moves along a straight line is given by $a = (2t-1)\ m/s^2$, where $t$ is in seconds. If $s = 1$ m and $v = 2$ m/s when t = 0, determine:

a) The particle's velocity and position when $t = 6$ s.

b) The total distance $d$ the particle travels during this time period.

$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v\Delta t$

**Solution:**

a) we know that $a = \frac{dv}{dt} \Rightarrow a\,dt = dv$ by integrating

$\Rightarrow \int_{t_0}^{t} a\,dt = \int_{v_0}^{v} dv$ but $t_0 = 0$, $v_0 = 2\,m/s$ (given)

thus $\int_0^t (2t-1)\,dt = \int_2^v dv \Rightarrow v\Big]_2^v = \frac{2t^2}{2} - t\Big]_0^t$

$\Rightarrow v - 2 = t^2 - t \Rightarrow \boxed{v = t^2 - t + 2}$ (m/s)

or we know also that $v = \frac{ds}{dt} \Rightarrow v\,dt = ds$ by integrating

$\Rightarrow \int_{t_0}^{t} v\,dt = \int_{s_0}^{s} ds$ but $t_0 = 0$, $s_0 = 1\,m$ (given) $\Rightarrow \int_0^t (t^2 - t + 2)\,dt = \int_1^s ds$

$\Rightarrow s - 1 = \frac{t^3}{3} - \frac{t^2}{2} + 2t \Rightarrow \boxed{s = \frac{t^3}{3} - \frac{t^2}{2} + 2t + 1}$ (m)

@ t = 6s $\Rightarrow v = (6)^2 - (6) + 2 = 32\ m/s \Rightarrow \boxed{v = 32\,m/s}$

$s = \frac{(6)^3}{3} - \frac{(6)^2}{2} + 2(6) + 1 \Rightarrow \boxed{s = 67\,m}$

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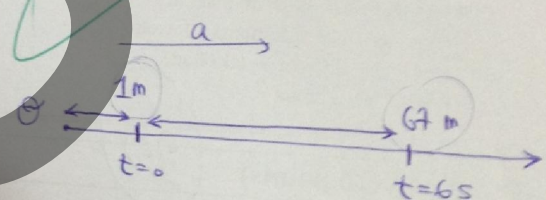
For the total distance traveled we have:

~~$d = v.t = 32 \times 6 = 192$ m~~

$\Delta S = S_{@6s} - S_{@t=0s}$

$= 67 - 1 = \underline{\underline{68 \text{ m}}}$

(According the the graph)



## Problem II (20 points)

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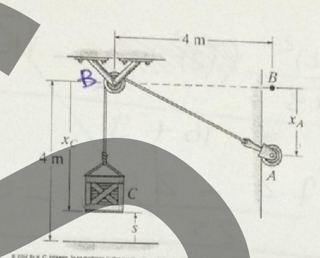
The crate $C$ is being lifted by moving the roller at $A$ downward with a constant speed of $v_A = 2\,m/s$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1$ m. When the roller is at $B$, the crate rests on the ground. Neglect the size of the pulley in the calculation.

***Hint:*** Relate the coordinates $x_c$ and $x_A$ using the problem geometry, then take the first and second time derivatives.




$l_{rope} = 4 + 4 = 8m$

**Solution:** Since we have a dependant motion we can say that:

length of the rope: $l = x_c + AB$ but $AB^2 = (4)^2 + (x_A)^2$ 5

$\Rightarrow AB^2 = 16 + x_A^2 \Rightarrow AB = \sqrt{16 + x_A^2} \Rightarrow \boxed{l = x_c + \sqrt{16 + x_A^2}}$ (1)

deriving (1) with respect to time: $\Rightarrow 0 = v_c + \frac{1}{\cancel{2}\sqrt{16 + x_A^2}} \cdot \cancel{2} x_A \cdot v_A$

$\Rightarrow \boxed{v_c = \frac{-x_A v_A}{\sqrt{16 + x_A^2}}}$ (2) 5

deriving a second time with respect to time $\Rightarrow$

$$a_c = -\left[\frac{\sqrt{16 + x_A^2}\cdot(v_A^2 + x_A a_A) - x_A \cdot v_A \cdot \frac{1}{2\sqrt{16 + x_A^2}} \cancel{2} x_A \cdot v_A}{16 + x_A^2}\right]$$

$$\Rightarrow \boxed{a_c = \frac{\frac{x_A^2 \cdot v_A^2}{\sqrt{16 + x_A^2}} - \sqrt{16 + x_A^2}\cdot(v_A^2 + x_A a_A)}{16 + x_A^2}}$$ (3) 5

for $s = 1m \Rightarrow x_c = 4 - 1 = 3m \Rightarrow AB^2 = (8-3)^2 = (4)^2 + x_A^2$

$\Rightarrow 5^2 = 4^2 + x_A^2$

$\Rightarrow x_A^2 = 25 - 16 = 9 \Rightarrow \boxed{x_A = 3m}$

in (2) $\Rightarrow v_c = \frac{-(3)(2)}{\sqrt{16 + (3)^2}} = \frac{-6}{5} = -1.2$ m/s $\downarrow$ = 1.2 m/s $\uparrow$

But $v_A = 2\,m/s$ = constant (given) thus

$a_A = 0$

(3) $\Rightarrow$ $$a_C = \frac{+\dfrac{(3)^2\cdot(2)^2}{\sqrt{16+9}} - \sqrt{16+9}\cdot\left((2)^2+0\right)}{16+9}$$

Solving $\Rightarrow$ $$a_C = \frac{7.2 - 20}{25} = -0.512\,m/s^2 \downarrow$$

thus $\boxed{a_C = 0.512\,m/s^2 \uparrow}$

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## Problem III (20 points)

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The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity $\dot{\theta} = 4\ rad/s$ and an angular acceleration $\ddot{\theta} = 8\ rad/s^2$ at the instant $\theta = 30^\circ$, determine the force $F$ of the arm guide on the particle and the normal support reaction $N$ of the circular path.

**N.B.:** Motion occurs in the horizontal plane.

***Hint:*** Find an expression for $r$ as a function of $\theta$.

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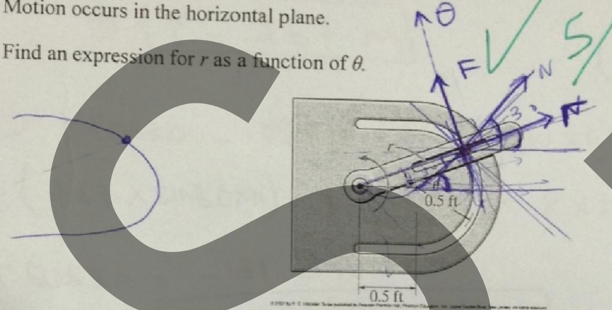
$\dot{\theta} = 4\ rad/s$

$\ddot{\theta} = 8\ rad/s^2$

$\theta = 30^\circ$

$\cos\theta = \frac{1}{r}$

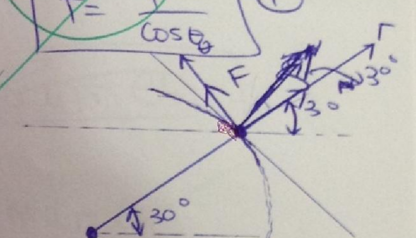
$r\cos\theta = 1$




**Solution:** Since the motion occurs in the horizontal plane thus the weigh is not included in the free body diagram

Refering to the geometry of the figure we have: $\boxed{r\cos\theta = 1}$

$\cos\theta = \frac{(0.5+0.5)}{r} \Rightarrow r = \frac{0.5+0.5}{\cos\theta} \Rightarrow \boxed{r = \frac{1}{\cos\theta}}$ ①




~~deriving~~ Newtons second Law:

$\Sigma F_\theta = m a_\theta$

$\Sigma F_r = m a_r$

but $a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Deriving ① with respect to time $\Rightarrow$ $\boxed{\dot{r}\cos\theta - r\sin\theta \cdot \dot{\theta} = 0}$ ②

deriving a second time w.r.t $\Rightarrow$

$\Rightarrow (\ddot{r}\cos\theta - \dot{r}\sin\theta \cdot \dot{\theta}) - (\dot{r}\dot{\theta}\sin\theta + r(\ddot{\theta}\sin\theta + \dot{\theta}\cos\theta \cdot \dot{\theta})) = 0$

$\Rightarrow \boxed{\ddot{r}\cos\theta - \dot{r}\sin\theta \cdot \dot{\theta} - \dot{r}\dot{\theta}\sin\theta - r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta = 0}$ ③

@ $\theta = 30 \Rightarrow \boxed{r = \frac{1}{\cos 30} = 1.1547\ m}$

in ② $\Rightarrow \dot{r}\cos 30 - 1.1547 \sin(30) \times 4 = 0$

$\Rightarrow \boxed{\dot{r} = 2.67\ m/s}$

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in ③ $\Rightarrow \ddot{r}\cos 30 - \cancel{r\sin\theta}\cdot(2.67 \times \sin 30 \times 4)$

$-(2.67 \times \sin 30 \times 4) - (1.1547 \times 8 \times \sin 30) - (1.1547 \times 16 \times \cos 30) = 0$

$\Rightarrow \ddot{r}\cos 30 = -31.298$

$\Rightarrow \boxed{\ddot{r} = +36.140\ m/s^2}$

$\Rightarrow a_r = +36.140 - (1.1547 \times 16) = \cancel{-84.6152}\ 17.6648\ ft/s^2$

$a_\theta = 30.5976\ ft/s^2$

$\Rightarrow \cancel{F = ma_\theta}$

$\Rightarrow \cancel{F = \frac{0.5}{32.2} \times 30.5976} = \boxed{0.475\ lb}$

$\sum F_r = ma_r \Rightarrow N\cos 30 = ma_r$

$\Rightarrow N\cos 30 = \frac{0.5}{32.2} \times +17.6648$

$\Rightarrow \boxed{N = +0.2741\ N}$

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$\sum F_\theta = ma_\theta \Rightarrow$

$F + N\sin 30 = ma_\theta$

$\Rightarrow \boxed{F = ma_\theta - N\sin 30 = \cancel{15.782}\ lb}$

$\underline{0.338\ lb}$

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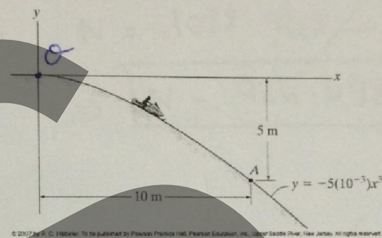
**Problem IV (20 points)** 20

The 200-kg snowmobile with passenger is traveling down the hill such that when it is at point *A*, it is traveling at 4 m/s and increasing its speed at 2 m/s². Determine the resultant normal force *N* and the resultant frictional force *f* exerted on the tracks at this instant. Neglect the size of the snowmobile.



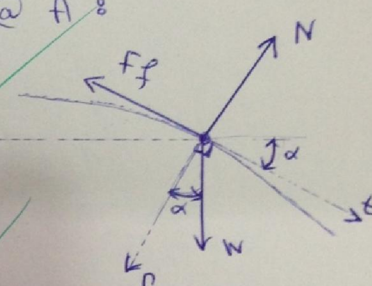

$V_A = 4\,m/s$

$a_{t} = 2\,m/s^2$

$m = 200\,kg$

**Solution:**

First let's draw the free body diagram @ A:




$\frac{dy}{dx} = -0.015x^2$ @ $x = 10$ $\Rightarrow \frac{dy}{dx} = -1.5 = \tan\alpha$

$\frac{d^2y}{dx^2} = -0.03x$ $\Rightarrow \boxed{\alpha = 56.309°}$

Newtons second law:

$\Sigma F_t = ma_t \Rightarrow -F_f + W\sin(56.309) = ma_t$

$\Rightarrow \boxed{F_f = W\sin(\cancel{56.309}) - ma_t}$ ① but $a_{tA} = 2\,m/s^2$ (given)

$\Rightarrow F_f = (200 \times 9.81 \times \sin 56.309) - 200(2) = 1232.46\,N$

$\Rightarrow \boxed{F_f = 1232.46\,N}$

$\Sigma F_n = ma_n \Rightarrow -N + W\cos(\cancel{56.309}) = ma_n = \frac{m \cdot V_A^2}{\rho}$

$\Rightarrow \boxed{N = W\cos\alpha - \frac{mV_A^2}{\rho}}$ ②

but $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \Rightarrow$ $\rho$ @ $x = 10\,m \Rightarrow \rho = \frac{\left[1 + (-1.5)^2\right]^{3/2}}{|-0.3|}$

$\Rightarrow \boxed{\rho = 19.53\,m}$

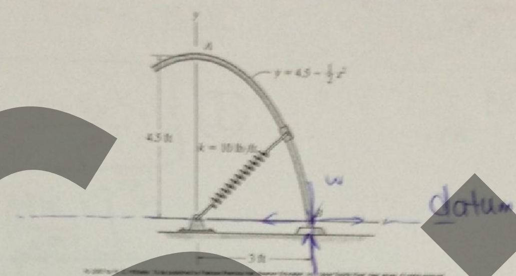
$$\Rightarrow \quad N = 200 \times 9.81 \times \cos(56.309) - \frac{200 \times (4)^2}{19.53}$$

$$\Rightarrow \quad N = 1088.348 - 163.85$$

$$\Rightarrow \quad \boxed{N = 924.498 \text{ Newtons}}$$

## Problem V (20 points)

The 2-lb collar has a speed of 5 ft/s at $A$. The attached spring has an unstretched length of 2 ft and a stiffness $k = 10$ lb/ft. If the collar moves over the smooth rod, determine its speed $v_B$ when it reaches point $B$, the normal force $N$ of the rod on the collar, and the rate of decrease in its speed.




$W = 2 lb$
$V_A = 5 ft/s$
$l_0 = 2 ft$
$K = 10$ lb/ft
$V_B = ?$

**Solution:** Since the collar is subjected to conservative force (No friction, weight and force of spring) thus we can applay the principle of conservation of energy between A and B:

$\Rightarrow \quad T_A + V_A = T_B + V_B$ ( Datu — pass in horizontaly trough B)

~~but $T_A = \frac{1}{2}mV_A^2 = 0.5 \times \frac{2}{32.2} \times (5)^2 = 0.77639$ Joules~~

~~$V_A = mgh_A = \frac{2}{32.2} \times 32.2 \times 4.5 = 9$ Joules.~~

~~$T_B = \frac{1}{2}mV_B^2 = \frac{1}{2} \times \frac{2}{32.2} V_B^2 = (0.03105\, V_B^2)$ Joules~~

~~$V_B = 0$ (datum)~~

~~$\Rightarrow 0.77639 + 9 = 0 + 0.03105\, V_B^2$~~

~~$\Rightarrow 0.03105\, V_B^2 = 314.239 \Rightarrow V_B = 17.72$ ft/s~~

$$\Rightarrow \boxed{\frac{1}{2}mV_A^2 + mgh_A + \frac{1}{2}ks_1^2 = \frac{1}{2}mV_B^2 + \frac{1}{2}ks_2^2} \quad (1)$$

$$\Rightarrow \left(\frac{1}{2} \times \frac{2}{32.2} \times (5)^2\right) + \left(\frac{1}{32.2} \times 32.2 \times 4.5\right) + \frac{1}{2}(10)(4.5-2)^2 = \frac{1}{2} \times \frac{2}{32.2} \cdot V_B^2 + \frac{1}{2}(10)(3-2)^2$$

$$0.77639 + 9 + 31.25 = 0.03105\, V_B^2 + 5$$

$$\Rightarrow \boxed{V_B = 36.364 \text{ ft/s}}$$

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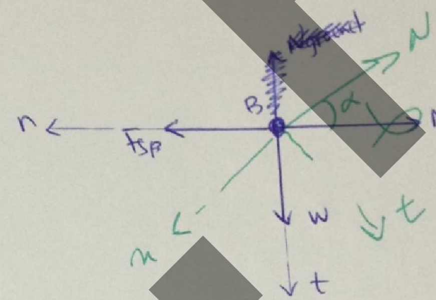
$\sum F_n = m a_n$

$\Rightarrow F_{sp} - N = m a_n = \frac{m v_B^2}{\rho}$

$\Rightarrow \boxed{N = F_{sp} - \frac{m v_B^2}{\rho}}$ (2)




F.B.D @ B

but $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$

$\Rightarrow \frac{dy}{dx} = -x \Rightarrow @\ x = 3\ ft \Rightarrow \frac{dy}{dx} = -3$

$\frac{d^2y}{dx^2} = -1$ $\qquad \frac{d^2y}{dx^2} = -1$

$\Rightarrow \rho = \frac{\left[1+(-3)^2\right]^{3/2}}{|-1|} = \boxed{31.622\ ft}$ ✓ 3/

$\Rightarrow N = k\Delta s - \frac{m v_B^2}{\rho} = 10\times(3-2) - \frac{\frac{2}{32.2}\times(36.34)^2}{31.622}$

$\Rightarrow N = 10 - 2.593 \Rightarrow \boxed{N = 7.406\ lb}$

$\sum F_t = m a_t \Rightarrow W = m a_t$

$W = m a_t \Rightarrow a_t = \frac{W}{m}$

$\Rightarrow a_t = \frac{2}{\frac{2}{32.2}} = 2\times\frac{32.2}{2} = \boxed{32.2\ ft/s^2}$

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